JOINT-OUTPUTS OF LAND AND WATER WASTES IN A LINEAR PROGRAMMING MODEL FOR AIR POLLUTION CONTROL

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A linear programming model has been developed for the St. Louis Airshed. This model is based on the simplifying assumption that ambient air quality goals can be achieved by reducing total emissions of each pollutant in an airshed to a predetermined allowable level for that pollutant. The reduction in emissions is obtained by instituting a least cost set of air pollution control method activity levels. A mathematical formulation of the model and the type of data used to characterize emission sources and the abatement technology is contained in an appendix to this paper. A more complete description of the model and some of the results have been reported elsewhere [5, 6, 7, 8, 9]. Although it was apparent that certain control methods for air pollution alter the flow of land, water, thermal, and other wastes, these interrelated pollutants were ignored in the original model.

In a 1969 article in the *American Economic Review,* Ayres and Kneese argue that the "primary interdependence between the various waste streams . . . casts into doubt the traditional classification of air, water, and land pollution as individual categories for purposes of planning and control policy." [1, p. 286] They warn that a partial equilibrium approach, "while more tractable, may lead to serious errors." [1, pp. 295, 6] This paper examines the nature of the errors which may have been introduced into the linear programming model by ignoring three specific external waste streams created by air pollution abatement measures. These are (a) liquid wastes, (b) landfill waste, and (c) thermal discharge to rivers. The nature and concentration of the contaminant in waste water or the type of solid waste that is landfilled is ignored; it is assumed that each external waste flow is homogeneous.

The solutions for four different models will be examined:

(I) a model for air pollution control in which the three external waste flows associated with the optimal solution are totaled but not constrained,

(II) as Model I, but with the provision that the resultant external waste flows be less-than-or-equal-to zero,

(III) the air pollution control model augmented to include net costs for reprocessing the joint-output waste flows,

(IV) as Model III, but including feed-backs on the air pollution model associated with the treatment of the external wastes.

This sequence of models, which is presented in mathematical form in the appendix, may be related to Figure 1. The graph, derived from the original model by parametric programming, is an isoquant for the given set of air quality standards for the St. Louis Airshed in 1975.[1] The goals can be achieved with varying combinations of market resources, z, and a non-market resource, w, the disposal capacity of the river system. The latter is measured in thousands of gallons of disposed liquid waste. Joint-outputs of landfill and thermal wastes are ignored in this diagram. If waste water output is unconstrained (Model I), the optimal solution is at point A, where total cost of air pollution abatement is minimized. If waste water output must be less-than-or-equal-to zero (Model II), the optimal solution is at B. If a shadow price for w is introduced, say \$.25 per thousand gallons (Model III), an isocost line is defined; in this particular example, the least cost solution is still at point A. If there are feed-backs between water treatment and air pollution abatement (Model IV), the isoquant shifts with the quantity of water treated and a solution cannot be graphed in two dimensions.

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[1]The non-economic, upward sloping segment of the isoquant (distinguished by dashes) may be questioned. If the technology in the model were to include possibilities for increasing liquid waste at zero cost (say, by reduced recirculation of scrubbing water), the isoquant would very likely contain a horizontal facet to the right of point A.

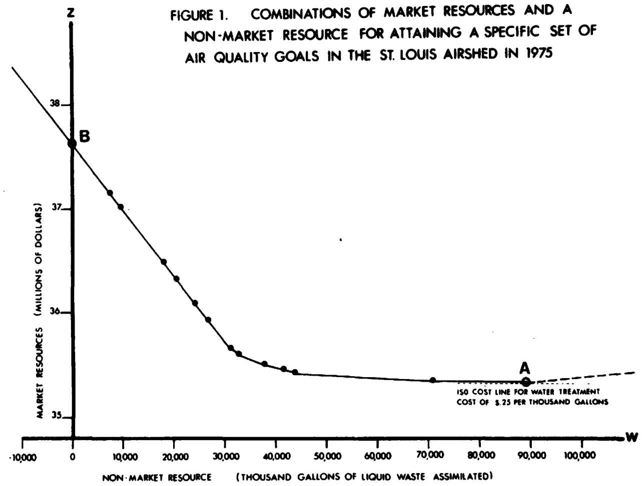

FIGURE 1. COMBINATIONS OF MARKET RESOURCES AND A NON-MARKET RESOURCE FOR ATTAINING A SPECIFIC SET OF AIR QUALITY GOALS IN THE ST. LOUIS AIRSHED IN 1975

The air quality goals used in the model are related, in part, to those adopted by the Missouri Air Conservation Commission. They are annual average ambient air concentrations of 5 ppm for carbon monoxide, 3.1 ppm for total hydrocarbons, .069 ppm for nitrogen oxides, .02 ppm for sulfur dixoide, and 75 $\mu g/m^3$ for particulates. The solution of each model includes non-zero activity levels for more than 50 out of 215 control methods. To facilitate comparison of the four solutions, the following will be examined:

a) total cost of air pollution abatement (This includes the market value of labor and materials, the depreciation of equipment, the opportunity cost of invested capital, the economic value of substitute fuels, less credits for recovered by-products.)
b) total cost of reprocessing the external wastes generated by air pollution abatement
c) the quantities of untreated external wastes
d) shadow prices for the external waste constraints
e) quantities of external wastes reprocessed
f) quantity of natural gas replacing coal in the least cost solution (The fluctuations in this quantity provide insight on what is happening in the model.)
g) marginal cost of abatement (the dual values) for sulfur dioxide and particulates.

The values for these indicators are listed in Table 1.

Model I: The Original Model

The solution of Model I indicates that the cost of achieving the air quality goals for the St. Louis Airshed in 1975 is \$35,000,000 plus 89,000 thousand gallons of disposed liquid waste, 450,000 tons of landfill waste, and 1,400,000 million Btu's of heat discharged to rivers. It should be noted these are the incremental costs of air pollution abatement and do not include the cost and waste outputs associated with the pre-regulation, or 1963 base-year level of air pollution control. The external waste flows which would be created are in addition to an estimated 435,000,000 thousand gallons of waste water, 865,000 tons of landfill waste, and 120,000,000 million Btu's of thermal discharge from all human activites unrelated to air pollution control in the St. Louis Airshed in 1975.

TABLE 1. AIR POLLUTION ABATEMENT MODELS

Indicators	I No Constraints on External Waste Streams (solid, liquid, thermal wastes)	II Less-than-or-equal-to zero Constraints on External Waste Streams	III Reprocessing Prices on the External Waste Outputs	IV Prices on the External Wastes and Feedbacks on the Model from Reprocessing
Cost of Air Pollution Abatement	$35,337,283.	$56,106,262.	$35,398,680.	$35,405,008.
Cost of Reprocessing the External Wastes	0	0	$514,832.	$517,013.
Thousand Gallons of Liquid Waste Generated	88,865	0	0	0
Tons of Solid Waste Generated	450,082	0	0	0
Million Btu's of Heat Discharged to Rivers	1,363,366	0	0	0
Shadow Price per Thousand Gallons of the Liquid Waste Constraint	0	$92.33	$.25	$.27
Shadow Price per Ton of the Solid Waste Constraint	0	$17.94	$1.40	$1.40
Shadow Price per Million Btu's of the Thermal Waste Constraint	0	$20.07	$.04348	$.04729
Thousand Gallons of Liquid Waste Treated	0	0	94,975	94,975
Tons of Solid Waste Reprocessed	0	0	539,474	538,892
Million Btu's of Thermal Waste Diverted	0	0	1,368,699	1,429,583
Optimal Quantity (in Millions of Cubic Feet) of Natural Gas Replacing Coal	14,193	72,948	14,288	14,370
Marginal Cost of Abatement for One Pound of Sulfur Dioxide	$.02193	$.06819	$.02193	$.02193
Marginal Cost of Abatement for One Pound of Particulates	$.07748	$.17357	$.09206	$.09208

TABLE 2. REPROCESSING METHODS FOR THE EXTERNAL WASTE FLOWS

	Primary, secondary, and tertiary treatment of municipal waste water	Combustion of processed refuse as supplementary power plant fuel	Hyperbolic natural draft cooling tower
Activity Unit	Thousand gallons	Ton of refuse	Thousand Kilowatt Hours
Cost per Activity Unit	$.25	$.80	$.20
Output of Liquid Waste	−1. thousand gallons	0	0
Output of Land Waste	.00045 tons	−.7 tons	0
Direct or Indirect Output of Thermal Waste	.046 million Btu	.1035 million Btu	−4.6 million Btu
Coal Burned at the Sioux Power Plant to Produce Required Electricity	.004 tons	.009 tons	.004 tons
Reduced Emissions of Carbon monoxide	−.0012 pounds	−.4027 pounds	−.0012 pounds
Reduced Emissions of Hydrocarbons	−.0005 pounds	−.1011 pounds	−.0005 pounds
Reduced Emissions of Nitrogen oxides	−.0472 pounds	−.1062 pounds	−.0472 pounds
Reduced Emissions of Sulfur dioxide	−.2511 pounds	1.5350 pounds	−.2511 pounds
Reduced Emissions of Particulates	−.0009 pounds	−.0022 pounds	−.0009 pounds

The incremental amount of liquid waste from air pollution abatement is an insignificant percentage of the estimated volume of waste water. This may be attributed to the fact that only five of the optimal air pollution control methods affect water pollution. The major contributor is the dolomite wet scrubbing process for power plant desulfurization, which involves a discharge of 70 gallons of waste water per ton of coal burned. However, it is possible that the present model does not include an adequate representation of scrubber-type control methods, so that the volume of waste water associated with an optimal solution for air pollution control may be understated.[2]

The solid waste that must be landfilled as a result of air pollution abatement represents more than a 50% increase in the projected landfill tonnage. This includes recovered, unsalvageable particulate matter and the solid waste from control methods which replace open burning or incineration with landfill disposal.[3]

The incremental thermal discharge is associated with the generation of 300 million kilowatt hours, the annual electrical requirements for the set of optimal control methods.[4] This represents approximately one percent of the projected electrical output of the utility power plants in the St. Louis Airshed in 1975. [6, pp. 239-303] It is assumed that there are .0046 million Btu's of heat discharged to the cooling water for each kilowatt-hour generated.[5]

[2]The insufficient representation of water using, air pollution control methods in this model may be attributed to the unavailability of data on local chemical processing industries, which would be major users of scrubbing water. As a consequence of this lack of data, a single source is proxy for all hydrocarbon chemical processing. Furthermore, it was assumed when the model was originally set up, that this source was initially equipped with scrubbers and that the optional control method would be an afterburner in combination with the scrubber. As a result of this simplification, a number of incremental-water-using control methods may be missing from the model.

[3]It should be emphasized that the external waste flows are joint-outputs of an optimal solution of a specific model. The air pollution regulations being enforced in St. Louis are more stringent with regard to open burning and incineration than the solution of the present model. As a result, the increase in land waste for the St. Louis Airshed as a result of air pollution control will be larger than the figure suggested here.

[4]The original model did not account for the additional coal combustion that would be required to generate this electricity. In the present study, only the incremental thermal pollution is considered. In a subsequent study, Model I was rerun to investigate the feedback effect of any additional, required coal combustion. Except for a 1% increase in total abatement cost, the optimal set of control methods and dual values was essentially unchanged.

[5]This is an average figure based on data in reference 15, p. 291.

Model II: The Original Model with Constraints on External Wastes

Model II incorporates the constraint that there be no incremental external waste flows as a consequence of controlling air pollution. Such a requirement, although stringent, is less binding than the stipulation sometimes expressed, that any control methods which reduce one waste flow while increasing another should never be used.[6] Model II permits the use of such air pollution control methods but requires that any incremental joint-wastes be offset within the technological framework of the model. The resulting solution indicates that air pollution abatement with zero external waste outputs would cost over $56 million in 1975. This solution corresponds to point B in the two dimensional Figure 1. The shadow prices of the external waste flows are the marginal costs associated with the constraints. If, for example, an incremental joint-output of one thousand gallons of liquid waste were allowed, the total cost of air pollution abatement would decline by $92.33.

For the most part, the waste flows are offset within the linear programming model by conversion of certain types of furnaces from coal to natural gas. An example is a control method which represents the conversion of travelling grate stokers with mechanical dust collectors to natural gas.[7] This eliminates 170 pounds of bottom ash and 40 pounds of collected fly ash per ton of coal burned. In addition, the retirement of the stoker and mechanical collector eliminates power consumption of about 12 kilowatt hours per ton of coal. In the case of travelling grate stokers that are equipped with wet scrubbers, conversion to natural gas reduces the output of waste water as well as bottom ash. It will be noted in Table 1 that because of the imposition of constraints on external wastes, the marginal costs of controlling sulfur dioxide and particulates are considerably higher for Model II than Model I.

Model III: The Simple, Augmented Model

The high shadow costs for the three external waste flows in Model II suggest that it would be inefficient to require that an air pollution control program involve no incremental external wastes. There are methods to reprocess equivalent quantities of these wastes that cost less than the

[6]For example, the dolomite wet scrubbing process for power plants received some criticism because in the process of eliminating sulfur dioxide and particulate air pollution, it adds some salt content to the discharged water.

[7]The air pollution coefficients for this control method, identified as Control Method 21F, are discussed in reference 8. A complete description of the priority of conversion, by type of stoker, is contained in reference 7.

shadow prices indicated. In Model III, three reprocessing methods are added to the model. Only the cost and output (waste flow reduction) of these methods are included in Model III, so that in effect, a single opportunity cost of reprocessing each waste stream is introduced. The estimated coefficients for the three reprocessing methods are presented in Table 2.

The cost of water purification depends on the nature of the waste content, and an accurate model would have a range of water treatment prices. For simplicity, the present model uses a single reprocessing method; primary, secondary, and tertiary treatment of municipal waste water.[8]

It is assumed here that a substantial quantity of solid waste can be used to generate electricity. The City of St. Louis, with partial support of a federal grant, is constructing pilot facilities to prepare municipal refuse for use as fuel by Union Electric Co. The project is based on a feasibility study prepared by Horner and Shifrin, Inc. [16] The cost for this reprocessing method is the estimated cost of preparing, transporting, and firing the refuse in the Labadie power plant, less the value of recovered heat and less the avoided costs of landfill disposal.

It is assumed that any land waste generated by air pollution abatement could be offset by the diversion of municipal waste from landfill to utilization as fuel. The land waste output for this reprocessing method is −.7 tons.[9] This is based on the assumption that the metallic and ash content, approximately 30% by weight, must still be buried. Actually, there is a good possibility that this residue will be recycled, in which case the land waste output coefficient should be −1.

The social anxiety over solid waste is based in part on (1) the mounting relative costs of pollution-free disposal, (2) the possibility that resources are being too rapidly depleted and that wastes should therefore be recycled,[10] and (3) concern that landfill limits the future use of land and frequently destroys so-called wastelands, which have an important ecological role. The use of solid waste as fuel is a partial answer to each of the above. A ton of prepared refuse replaces .4 tons of coal, thereby conserving a resource[11] and slowing the rate of environmental conversion by strip mining.[12]

The reprocessing method for thermal pollution is a hyperbolic natural draft cooling tower for the Labadie power plant. It is assumed here that the problem of thermal pollution is eliminated if the waste heat from power production is discharged to the atmosphere.[13]

If the unit cost of reprocessing is divided by the output coefficient, the shadow price for the external waste constraint, which is a part of the computer program output, can be independently determined.[14] For thermal pollution, the shadow price, using data from Table 2, is $.20/4.6 = $.04 per million Btu's. The shadow cost for solid waste is probably an upper bound. It is likely that there are situations where producers could effect substitutions resulting in reduced solid waste for less than $1.40 per ton.

The cost of reprocessing the external waste joint-outputs in the solution of Model III exceeds $500,000. By imposing costs on what were free environmental services in Model I, some substitution away from the external waste outputs would be expected in Model III. This substitution is not readily apparent in Table 1; in fact, the quantities of liquid and thermal wastes reprocessed are somewhat greater than their outputs in Model I. However, the land waste joint-output of air pollution abatement drops to 378,000 tons. (Reprocessing 539,474 tons of solid waste eliminates 378,000 tons of land waste, assuming a metallic and ash content of 30%). Valuing the joint-outputs at their shadow prices in Table 1, there is a decline in the value of the three joint-outputs of air pollution abatement in Model III.

The reduction in solid waste output in Model III can be traced to a change in the optimal set of control methods. A control method in which on-site open burning is replaced by sanitary landfill disposal is no longer optimal in Model III, as it was in Model I. The cost imposed on incremental landfill waste now offsets the air pollution benefits for this particular control method.

The marginal cost of controlling particulate matter is higher in Model III than Model I. The optimal solution, as a consequence, includes three additional control methods for abatement of particulates. It is of interest that two of these control methods do not increase solid wastes; they apply to grain processing and cement manufacturing, where the recovered particulate matter is saleable output.

The optimal quantity of natural gas increases very little in the model with costs on external wastes over that in Model I. In view of the fact that conversion to gas reduces the external wastes (note the great increase in natural gas usage in Model II), one might expect an increased emphasis on this fuel in Model III. However, it appears to be more economical to reduce external wastes by reprocessing them, rather than by converting additional boilers to natural gas.

The fact that there are some changes in the optimal set of air pollution control methods, as a consequence of imposing costs on what were formerly free services, suggests that the cost of air pollution abatement will be higher in Model III than Model I. Table 1 indicates that the cost increase is $60,000. This relatively small increase suggests that the solution of Model III corresponds, figuratively, to a tangency solution close to point A in Figure 1.

Model IV: The Augmented Model with Feedbacks

In Model IV, the feedbacks associated with reprocessing the external wastes are introduced. This adds more complexity to the model than the simple inclusion of opportunity costs. In the case of water treatment, some solids remain after digestion of sludge, and there is additional air pollution and thermal waste associated with the electric power requirements. It is estimated that the power to reprocess one thousand gallons of water requires the additional combustion of .004 tons of coal at the Sioux power plant.[15]

There is a net increase in electric power requirements associated with the grinding and shredding to reprocess solid wastes. However, the increased air pollutants reflect not only the incremental power generation at the Sioux plant but also the difference in emissions when four-tenths of a ton of coal are replaced in the Labadie power plant by a ton of prepared refuse. In the case of sulfur dioxide, the low sulfur content of refuse as compared to coal results in this reprocessing method being, on balance, a

[8]The coefficients for this reprocessing method are based principally on data in references 11 and 14. It is by no means clear that this is an optimal reprocessing method; Kardos, for example, suggests that municipal waste water should receive only primary and secondary treatment and then be recycled for use as enriched irrigation water for farmlands. See reference 4.

[9]Note that coefficients which indicate a reduction in air pollution are positive whereas those indicating a reduction in the external wastes are negative.

[10]The contention of increasing resource scarcity is examined and challenged by Barnett and Morse. See reference 2, chapters 1, 8, and 9.

[11]It is conceivable that the present value of prepared municipal refuse is *more* valuable than coal. Hart advocates that municipal refuse be composted and utilized to maintain and improve agricultural land. See reference 3, pp. 29-32.

[12]Krutilla suggests, in effect, that there is a socially optimal rate of environmental conversion for the production of consumption goods. See reference 10, p. 785.

[13]There may be serious ecological problems associated with the extensive evaporation of water from wet cooling towers. More research on this subject is needed, if cooling towers are to be used on a large scale basis. The coefficients for this reprocessing method are based on data in reference 15, p. 302.

[14]This is not quite true for the solid waste reprocessing method. The base level of pollution control in the St. Louis Airshed included the burning of approximately 5% of landfill waste. [6, p. 334] The $.80 cost for the reprocessing method is based on a credit for the cost of sanitary landfilling, which is more expensive than landfilling in which part of the bulk is reduced by burning. The way the model is set up, when landfill is diverted to fuel, there is a corresponding elimination of the 5% open burning. In effect, the cost of the reprocessing method includes, at the margin, an additional $.72, the cost to eliminate the open burning less $.54 which is the value of the foregone pollutants from landfill burning times the duals of the pollutants. The shadow price of the land waste constraint is therefore,

$$(\$.80 + \$.72 - \$.54)/(.7) = \$1.40.$$

[15]The incremental air pollution associated with the combustion of .004 tons of coal at the Sioux power plant is actually more than the quantities indicated in Table 2. The original and subsequent models incorporate a 41% reduction factor for emissions from stacks 600 feet high. This correction to "effective" emissions is based on data in reference 12.

method of reducing sulfur dioxide. The decrease is relatively small, however, because the refuse is burned in combination with coal, and a stack gas desulfurization process is simultaneously optimal.

The added pumping power and reduced turbine efficiency associated with the hyperbolic natural draft cooling tower involve increased power generation of one percent of plant capacity. [15, p. 302] Assuming that one ton of coal generates 2500 kilowatt hours, this is equivalent to .004 tons of coal combustion for each thousand kilowatt hours of electrical output in a power plant equipped with the cooling tower.

The major feedback effect is the additional power required for the three reprocessing methods. This results in additional combustion of 6600 tons of coal at the Sioux power plant. The consequent addition to the burden of air pollution control is reflected in the slightly higher cost of air pollution abatement for Model IV as compared to Model III.[16]

The cost of reprocessing external wastes increases in Model IV, mainly because the power requirements for the additional air pollution abatement activity increases the net quantity of thermal discharge. This increase in the cost of reprocessing external wastes is moderated by a decline in the joint output of solid waste; this decline is a by-product of the increase in natural gas replacing coal for air pollution abatement in Model IV.[17]

It is somewhat surprising that the relatively substantial feedbacks in Model IV produced very little change in the optimal solution. This may be due to the above observations that some of the feedbacks tended to offset each other. The marginal costs of sulfur dioxide and particulate abatement are virtually unchanged in Model IV, and a comparison of optimal control methods for Models III and IV indicate that with one minor exception, they are identical.

Conclusions

The reader is cautioned that the results in this paper are based on a least cost solution of a specific model for a single airshed, given a particular set of air quality goals. Any general conclusions based on these results must be considered in the light of this limitation.

The solution of Model I indicates that air pollution abatement is likely to generate alternative waste flows. The most significant of these was found to be incremental land wastes, representing more than a 50% increase in the projected landfill tonage in the St. Louis Airshed in 1975.

To arbitrarily prohibit incremental waste joint-outputs would increase substantially the cost of air pollution abatement; this is borne out by the results in Model II. A conclusion for policy making is that air pollution control strategy should take into account the possibilities for reprocessing external waste outputs.

The Ayres and Kneese warning of serious errors in a partial equilibrium approach may be tentatively challenged in view of the results for Model III. The increase in total cost of the optimal air pollution control methods, when external wastes are priced, is barely .2%, while the additional costs for reprocessing are less than 2% of total costs.

However, the models which incorporate the reprocessing methods are an improvement over the original model. The solid waste problem that is intensified when open burning or incineration is prohibited is given quantitative significance in the augmented models. The solutions for Models III and IV give some preference for air pollution control methods in which recovered particulate matter is recycled. In addition, the problem of relatively minor external wastes associated with certain sophisticated control methods, such as the dolomite wet scrubbing process for power plants, is put into proper perspective.

An unexpected finding was that, while conversion from coal to natural gas is of considerable economic value for air pollution abatement, its marginal value in reducing the external wastes was small. It costs less to reprocess incremental wastes than to reduce them by further conversions to natural gas. While this finding can hardly be generalized, it does serve to illustrate how more fully quantified models can avoid errors. On the basis of Model I and an intuitive weighting of the secondary benefits of waste reduction for natural gas, one might have been inclined to arbitrarily augment the optimal quantity of natural gas.

The similarity in results for Models III and IV suggests that the complicated feedbacks for the reprocessing methods might be ignored in future models, and simple opportunity costs for the external wastes suffice. However, considerably more environmental planning is necessary to determine the proper opportunity costs to be used.

Although the present model is a long way from the general equilibrium, total environment model envisaged by Ayers and Kneese, it is an advance beyond the isolated air pollution model, which these authors have challenged. [1]

MATHEMATICAL APPENDIX

There are M pollution sources, $s_1, s_2, \ldots, s_M$, in an airshed. Table 3 pertains to one such source; the combustion of refuse in flue-fed incinerators. It is projected that 34,000 tons of refuse will be burned in this type of incinerator in the St. Louis Airshed in 1975 [6, p. 318]. Based on the emission factors in the first column of Table 3, this source would generate 918,000 pounds of carbon dioxide, 68,000 pounds of hydrocarbons, etc. in that year.

The model contains N variables, $x_1, x_2, \ldots, x_N$, which represent the activity levels of air pollution control methods. These variables are characterized by cost, source (or input), and pollution abatement coefficients such as those illustrated in Table 3. Because of considerable variation in operating rates and types of refuse burned in flue-fed incinerators, the cost and pollutant coefficients for this particular source are crude averages at best. Although this is a relatively minor source of pollution in the St. Louis Airshed and a particularly difficult one to quantify as well, it is useful for illustrating the model.

The dollar cost for any set of control method activity levels is

$$1)\qquad \sum_{j=1}^{N} c_j x_j \ ,$$

where c_j is the unit cost of control method j. For example, the \$2.80 unit cost of converting from incineration to landfill disposal represents the capital and labor expended to collect and dispose of a ton of refuse in a sanitary landfill, less the avoided costs of incineration. A limitation of the model is the assumption of constant costs; *i.e.* that unit cost, c_j, is independent of the corresponding activity level, x_j.

The source, or input, constraint precludes the sum of activity levels of control methods from exceeding the magnitude of the source for which they are defined. Thus the combined activity levels of the four control methods in Table 3 cannot exceed 34,000 tons in the 1975 solution. This constraint is generalized by the equations,

$$2)\qquad \sum_{j=1}^{N} a_{ij} x_j \leqslant s_i \qquad\qquad (i = 1, 2, \ldots, M) \ ,$$

where a_{ij} is unity when control method j is defined for source i and zero otherwise.

[16]This cost reflects not only the increased air pollution control activity for the Sioux power plant but for other sources in the airshed as well. The model is based on an assumption of maximum allowable total emission flows for each pollutant in the airshed, and since the collection efficiencies of the optimal controls for the Sioux plant are less than 100%, the residual incremental emissions must be offset by increased abatement activity by other sources. In Model IV, this is accomplished by the conversion of additional boilers from coal to natural gas, as evidenced by the increase in the natural gas indicator in Table 1. The net increase in abatement cost would be at least \$20,000 more than shown, were it not for the offsetting credit from the reduction in sulfur dioxide associated with the burning of prepared municipal refuse in place of coal.

[17]This reduction of solid waste joint-output would account for the fact that the shadow price for the solid waste constraint does not increase in Model IV, as it does for the liquid and thermal constraints. These rises reflect the additional costs for the feedbacks associated with the reprocessing methods.

TABLE 3. COEFFICIENTS CHARACTERIZING A SET OF AIR POLLUTION CONTROL METHODS FOR FLUE-FED INCINERATORS

(the control method activity unit is one ton of refuse throughput controlled)*

Emissions and Waste per ton of refuse burned, without air pollution control	Description and symbol of the coefficient	Wet scrubber	Afterburner	Combination of wet scrubber and afterburner	Substitution of landfill disposal for incineration
	Abatement cost per ton of refuse controlled (c_j)	\$4.00	\$3.50	\$7.50	\$2.80
	Refuse throughput per unit of control method activity (a_{ij})	1. ton	1. ton	1. ton	1. ton
27. pounds	Abatement of carbon monoxide (b_{pj})	0 pounds	27. pounds	27. pounds	27. pounds
2. pounds	Abatement of hydrocarbons (b_{pj})	0 pounds	2. pounds	2. pounds	2. pounds
.3 pounds	Abatement of nitrogen oxides (b_{pj})	0 pounds	−9.7 pounds	−9.7 pounds	.3 pounds
.2 pounds	Abatement of sulfur dioxide (b_{pj})	0 pounds	0 pounds	0 pounds	.2 pounds
28. pounds	Abatement of particulates (b_{pj})	23.8 pounds	21.0 pounds	26.95 pounds	28. pounds
	Waste water generated per activity unit (d_{kj})	.43 thousand gallons	0 thousand gallons	.43 thousand gallons	0 thousand gallons
.235 tons of landfill ash	Land waste generated per activity unit (d_{kj})	0 tons	0 tons	0 tons	.765 tons
	Thermal discharge associated with electrical requirements of the control method (d_{kj})	.0184 million Btu	.0166 million Btu	.035 million Btu	0 million Btu

*Sources: Duprey, R. L., *Compilation of Air Pollutant Emission Factors,* N.A.P.C.A., Durham, N. C., 1968, pp. 9, 10.

Kaiser, E. R., et. al. "Modifications to Reduce Emissions from a Flue-Fed Incinerator," *A.P.C.A.J., 10,* June 1960, Table V, p. 190.

Kaiser, E. R., "Refuse Reduction Processes" in *Proceedings, The Surgeon General's Conference on Solid Waste Management for Metropolitan Washington, July 19-20, 1967,* U.S.P.H.S., Cincinnati, Ohio, p. 98.

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Control Techniques for Particulate Air Pollutants, N.A.P.C.A., Washington, D. C., 1969, p. 165.

Reference 6, pp. 317-323; reference 15, p. 291.

Associated with each control method is a set of pollutant abatement coefficients. For example, the particulate coefficients contained in Table 3 are based on assumed reduction efficiencies of 85% for the wet scrubber, 75% for the afterburner, 85% plus 75% X 15% for the two control methods combined, and 100% for landfill disposal.[18] The negative coefficients shown in the table indicate that *additional* nitrogen oxides are formed in the more intense heat of the afterburner. The inequality,

$$3) \quad \sum_{j=1}^{N} b_{pj}x_j \geq r_p \qquad (p = 1, 2, \ldots, P),$$

where b_{pj} is the reduction in pounds of pollutant p obtained with a unit of control method j activity, requires total pollution abatement for each of P pollutants to be no less than some specified quantity.

The required reduction, r_p, for any pollutant is the excess of anticipated annual emissions of that pollutant in the airshed over an allowable flow. The anticipated emission flows for 1975 were calculated by projecting each of 94 pollution source magnitudes (these include 10 categories of transportation, 7 classifications of power plants, 32 types of stationary fuel burning installations, 9 refuse burning activities, and 36 industrial processes) to the year 1975 and multiplying each quantity times a corresponding set of emission factors, such as those illustrated in the first column of Table 3.

The concept of an allowable flow is based on a simplifying assumption by Zimmer and Larsen that the annual average ambient air concentration of a pollutant at some central monitoring station, less the background concentration, is directly proportional to total emissions of that pollutant in the airshed [13]. The allowable flows in the present model are calculated from air quality goals in Table 4 and from data for 1963 (or 1964 where 1963 data are not available) according to the formula in a footnote to the table.

It is a limitation of this model that all sources are considered equivalent regardless of their location in the airshed. Models are now being developed which relate emissions at specific locations in an airshed to ambient air concentrations at various receptor stations. Such models, incorporating meteorological characteristics and atmospheric chemical interactions, should eventually provide more definitive economic solutions than does the present model.

The K external waste outputs associated with a set of air pollution control method activity levels are given by the equations,

$$4) \quad \sum_{j=1}^{N} d_{kj}x_j = w_k \qquad (k = 1, 2, \ldots, K),$$

where d_{kj} represents the quantity of waste k associated with one unit of control method j activity. Thus the wet scrubber in Table 3 generates .43 thousand gallons of waste water per ton of refuse throughput, and an additional power requirement of 4 kilowatt hours for the scrubber system involves an incremental thermal discharge of 18,400 Btu to the cooling waters at the power plant. Because incineration generates approximately .235 tons of ash for landfill disposal, the incremental solid waste when the incinerator is discontinued is less than one, or an estimated .765 tons.

[18]The latter control method involves emissions associated with the collection and transport of refuse. Unfortunately, these were neglected when the model was set up.

The complete linear programming model, with the non-negative constraint, has the form

$$\text{Minimize} \quad \sum_{j=1}^{N} c_jx_j$$

$$5) \quad \text{Subject to} \quad \begin{aligned} &\sum_{j=1}^{N} a_{ij}x_j \leq s_i && (i = 1, \ldots, M) \\ &\sum_{j=1}^{N} b_{pj}x_j \geq r_p && (p = 1, \ldots, P) \\ &\sum_{j=1}^{N} d_{kj}x_j = w_k && (k = 1, \ldots, K) \\ &x_j \geq 0 && (j = 1, \ldots, N). \end{aligned}$$

In Model I of this paper, the w_k are unconstrained variables and the least cost combination of control methods is found which satisfies the remaining right hand side constraints, including the values of r_p from the final column of Table 4. In Model II, the w_k are set equal to zero.

The augmented models, in which the external wastes are reprocessed, contains K additional activity variables, one for each of the external waste outputs. The augmented formulation is

$$\text{Minimize} \quad \sum_{j=1}^{N} c_jx_j + \sum_{j=N+1}^{N+K} c_jx_j$$

$$6) \quad \text{Subject to} \quad \begin{aligned} &\sum_{j=1}^{N} a_{ij}x_j - \sum_{j=N+1}^{N+K} a_{ij}x_j \leq s_i && (i = 1, \ldots, M) \\ &\sum_{j=1}^{N} b_{pj}x_j - \sum_{j=N+1}^{N+K} b_{pj}x_j \geq r_p && (p = 1, \ldots, P) \\ &\sum_{j=1}^{N} d_{kj}x_j - \sum_{j=N+1}^{N+K} d_{kj}x_j = 0 && (k = 1, \ldots, K) \\ &x_j \geq 0 && (j = 1, \ldots, N) \end{aligned}$$

In Model III, the a_{ij} and b_{pj} for j = N+1, ..., N+K, and the non-negative d_{kj} for j = N+1, ..., N+K, are set equal to zero. In Model IV, the feedback relationships for the K reprocessing methods are introduced. For the liquid waste reprocessing method, the increased coal consumption at the Sioux power plant is expressed by the coefficient,

$$7) \quad a_{ij} = .004,$$

the increased hydrocarbon emissions, by the coefficient

$$8) \quad b_{pj} = -.0005,$$

and the increased solid waste, by the coefficient

$$9) \quad d_{kj} = .00045.$$

TABLE 4. ALLOWABLE ANNUAL EMISSION FLOWS, ANTICIPATED FLOWS IN THE ST. LOUIS AIRSHED IN 1975, AND REQUIRED REDUCTIONS*

Pollutant	Emissions in 1963 in million pounds	Annual average ambient air concentration in 1963 (or 1964)	Annual average ambient air quality goal	Background concentration	Allowable annual flow in million pounds	Anticipated emission flow in million pounds in 1975	Required abatement in million pounds in 1975
	f_p^0	q_p^0	q_p^*	b_p	f_p^*	f_p	r_p
carbon monoxide	2920	6.3 ppm	5. ppm	0	2335[a]	4200	1865
hydrocarbons	995	3.1 ppm	3.1 ppm	1.5 ppm	995	1520	525
nitrogen oxides	305	.069 ppm	.069 ppm	0	305	415	110
sulfur dioxide	1180	.059 ppm	.02 ppm	0	400	1390	990
particulates	300	128 $\mu g/m^3$	75 $\mu g/m^3$	31 $\mu g/m^3$	135	300	165

Sources: Reference 6, pp. 445-457; reference 18, p. 24; also, *Air Quality Data from the National Air Sampling Networks and Contributing State and Local Networks, 1964-1965*, U.S.P.H.S., Cincinnati, Ohio, 1966, pp. 4, 25.

Formulas: $f_p^* = f_p^0 \, (q_p^* - b_p) / (q_p^0 - b)$

$r_p = f_p - f_p^*$

[a]The allowable flow for carbon monoxide is based on the concentrations, $q_p^0 = 75$ ppm and $q_p^* = 60$ ppm, which represent maximum one hour concentrations in traffic. Although the measures for carbon monoxide were subsequently changed to the above annual averages, resulting in a slightly different value for f_p^*, the original allowable level was used.

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